Diophantine Analysis and Related Fields 2013

Dates: From Tuesday 8 January to Wednesday 9 January, 2013

Location: Room 142, 4th Floor, Building 1, Suruga-dai Campus, College of Science and Technology (Rikou-Gakubu), Nihon University

Organizers: Shigeki AKIYAMA (Tsukuba University), Masaaki AMOU (Gunma University), Masanori KATSURADA (Keio University), Takao KOMATSU (Hirosaki University), Ryotaro OKAZAKI (Doshisha University), Noriko HIRATA-Kohno (Nihon University/local organizer)

Program

Tuesday 8 January

9:45-9:50	Opening
9:50-10:50	Shigeki AKIYAMA (Tsukuba University) Height reducing problem of algebraic numbers
11:00-12:00	Tünde KOVÁCS (University of Debrecen) Perfect powers in arithmetic progressions
12:00-14:00	Lunch break
14:00-14:50	Koichi KAWADA (Iwate University) Relations between densities of exceptional sets in Waring's problem
15:00-15:40	Hajime KANEKO (Nihon University, JSPS) Transcendence of the values of power series at certain algebraic points
16:00-16:40	Takafumi MIYAZAKI (Tokyo Metropolitan University, JSPS) The Diophantine equation $(an-1)^x + n^y = (an+1)^z$
16:50-17:30	Makoto KAWASHIMA (Osaka University) Irrationality of the values of the Lerch function $\Phi_s(x,z)$
17:40-18:10	Masaru ITO (Tokyo Institute of Technology) Optimization for lattices and Diophantine approximations
18:20-19:30	Reception party (at Cafeteria, 2nd Floor, Building 1)

Wednesday 9 January

9:50-10:40	Yu YASUFUKU (Nihon University) Exponential Diophantine Equations and Dynamical Mordell–Lang
10:50-11:50	Thomas TUCKER (University of Rochester) Integral points in orbits and the Bang-Zsigmondy problem
11:50-13:50	Lunch break
13:50-14:40	Yoshinori HAMAHATA (Ritsumeikan University) Values of Dedekind sums for function fields
14:50-15:40	Hiroshi MIKAWA (Tsukuba University) Large gaps between Hardy-Littlewood numbers
16:00-16:40	Ken KAMANO (Osaka Institute of Technology) Analytic continuation of the Lucas zeta function
16:50-17:30	Takao KOMATSU (Hirosaki University) Sums of products of Cauchy numbers, including generalized poly-Cauchy numbers
17:30-17:35	Closing

Supported partly by the Japan Society for the Promotion of Science (Next Program GR087, Grant-in-Aid for Scientific Research No. 22540005, No.23740033) and by College of Science and Technology, Nihon Universty.

Diophantine Analysis and Related Fields 2013

held at College of Science and Technology, Nihon University

Abstract of Talks

Tuesday 8 January, 2013

• 9:50-10:50 Shigeki AKIYAMA (Tsukuba University)
Title: Height reducing problem of algebraic numbers

Abstract: An algebraic number α has height reducing property (HRP), if there exists a finite set F such that $\mathbb{Z}[\alpha] = F[\alpha]$. Here $F[\alpha]$ is the values at α of polynomials having coefficients in F. Clearly if α has HRP, then it can not have conjugates of modulus less than one. We wish to characterize algebraic numbers with HRP and give a result which is 'almost' if and only if. In fact, we show that HRP implies that α is expanding, or all conjugates are on the unit circle. Conversely if α is expanding or a root of unity then it satisfies HRP. The remaining case is that α is an algebraic non-integer, whose all conjugates have modulus one. In this case, we can show a criterion which works in most of the cases, in particular, when the degree is not greater than six. So our natural expectation is that HRP is valid for all the remaining cases. The last part requires Mahler's version of Kronecker's approximation theorem and theory of linear forms in logarithms. Finally we shall present interesting concrete examples found by LLL algorithm, that all our methods do not apply. This is a joint work with Toufik Zaimi.

• 11:00-12:00 Tünde KOVÁCS (University of Debrecen)
Title: Perfect powers in arithmetic progressions

Abstract: A celebrated theorem of Erdős and Selfridge states that the product of consecutive positive integers is never a perfect power. A natural generalization is the Diophantine equation

$$x(x+d)\dots(x+(k-1)d) = by^n \tag{1}$$

in non-zero integers x, d, k, b, y, n with $\gcd(x, d) = 1$, $d \geq 1$, $k \geq 3$, $n \geq 2$ and $P(b) \leq k$. Here $P(u)$ stands for the largest prime divisor of a non-zero integer u, with the convention $P(\pm 1) = 1$.
By a conjecture of Erdős, equation (1) has no solutions in positive integers when $k > 3$ and $b = 1$. In other words, the product of k consecutive terms of a primitive positive arithmetic progression with $k > 3$ is never a perfect power. The conjecture of Erdős has recently been verified for certain values of k in a more general form.
In the talk I present the best known results concerning the different values of the power n and give some insight into the methods of the proofs. I speak about the case $n = 5$ in more detailed. I explain why this case is special and present extensions of the previous results due to Hajdu and myself. Apparently, for $n = 5$ elliptic curves are not applicable. We show that in this case the Chabauty method (both the classical and the elliptic versions) can be applied very efficiently. I finish the talk with some remarks on current work related to the topic.

• 14:00-14:50 Koichi KAWADA (Iwate University)
Title: Relations between densities of exceptional sets in Waring's problem

Abstract: When $\mathcal{A}$ and $\mathcal{B}$ are sets of natural numbers, let $\mathcal{A}+\mathcal{B}$ denote the set of all the sums $a+b$ for $a \in \mathcal{A}$ and $b \in \mathcal{B}$, and $\overline{\mathcal{A}}$ be the complement of $\mathcal{A}$ within $\mathbb{N}$. Then, generally speaking, the subject of this talk is relation between the densities of $\overline{\mathcal{A}}$ and $\overline{\mathcal{A}+\mathcal{B}}$. We present concrete results on such relations when $\mathcal{B}$ is, for example, the set of squares or cubes, and consequently establish a new estimate for the density of numbers that cannot be written as the sum of six cubes, amongst others. This is joint work with Trevor D. Wooley at University of Bristol.

• 15:00-15:40 Hajime KANEKO (Nihon University, JSPS)
Title: Transcendence of the values of power series at certain algebraic points

Abstract: Bailey, Borwein, Crandall, and Pomerance estimated lower bounds on nonzero digits of binary expansions of algebraic irrational numbers. As applications, they gave criteria for transcendence of the values of power series at certain rational points. Recall that β-expansions are generalizations of the expansions of real numbers in integral base b. In this talk we consider β-expansions of algebraic numbers in the case where β is a Pisot or Salem number. As applications, we give new criteria for transcendence of the values of power series at certain algebraic points.

• 16:00-16:40 Takafumi MIYAZAKI (Tokyo Metropolitan University, JSPS)
Title: The Diophantine equation $(an-1)^x + n^y = (an+1)^z$

Abstract: We solve the exponential Diophantine equation

$$(an-1)^x + n^y = (an+1)^z$$

in positive integers x, y and z, where a and n are fixed positive integers such that a is odd and n is even. The result generalizes the work for $a = 1$ due to B. He and A. Togbé ([1]). Also, as a corollary, we can completely solve the exponential Diophantine equation

$$b^x + 2^y = (b+2)^z$$

in positive integers x, y and z, where b is a fixed odd positive integer. This generalizes a result given by N. Terai ([2]). This is a joint work with A. Togbé (Purdue University).

References

[1] B. He and A. Togbé, The exponential Diophantine equation $n^x + (n+1)^y = (n+2)^z$ revisited, *Glasgow Math. J.* **51** (2009), 659–667.

[2] N. Terai, Applications of a lower bound for linear forms in two logarithms to exponential Diophantine equations, *Acta Arith.* **90** (1999), 17–35.

• 16:50-17:30 Makoto KAWASHIMA (Osaka University)
Title: Irrationality of the values of the Lerch function $\Phi_s(x, z)$

Abstract: Let s be a positive integer. E. M. Nikišin proved for a certain $z \in \mathbb{Q}$ the linear independence of $s+1$ numbers $1,\ F(1/z,1)\cdots, F(1/z,s)$, where $F(1/z,s) = \sum_{m=1}^{\infty} \dfrac{z^{-m}}{m^s}$. In our talk, we show a generalization of the above result of Nikišin in the case of the Lerch function $\Phi_s(x,z) = \sum_{m=0}^{\infty} \dfrac{z^{m+1}}{(m+x)^s}$.

• 17:40-18:10 Masaru ITO (Tokyo Institute of Technology)
Title: Optimization for lattices and Diophantine approximations

Abstract: Let $f(z)$ be an entire function in $\mathbb{C}$ with $f(\mathbb{N}) \subset \mathbb{Z}$. In 1915, G. Pólya proved that $f(z)$ is a polynomial provided that the order of growth of $f(z)$ is bounded by $< \log 2$. In this talk, we consider an entire function being almost integer-valued at positive integers and we try to optimize a growth condition such that $f(z)$ turns out to be a polynomial. Our argument relies on computational optimization for lattices combined with a method of Diophantine approximations.

Wednesday 9 January, 2013

• 9:50-10:40 Yu YASUFUKU (Nihon University)
Title: Exponential Diophantine Equations and Dynamical Mordell–Lang

Abstract: Inside a semiabelian variety G, a subvariety X and a finitely-generated subgroup $\Gamma \subseteq G(\mathbb{C})$ intersect in a very special form, namely a finite union of cosets of subgroups of Γ. This is the so-called Mordell–Lang conjecture, proved by Faltings and Vojta. A natural dynamical analog replaces Γ by the orbit of a point P under a finitely-generated commutative semigroup S of endomorphisms on G. We will show that this dynamical Mordell–Lang analog is utterly false. In fact, we will show that every exponential Diophantine equation with coefficients and bases in the ring of algebraic integers can occur as *the* intersection condition, if we choose G, S, P, and X appropriately. This is a joint work with Thomas Scanlon (UC Berkeley).

• 10:50-11:50 Thomas TUCKER (University of Rochester)
Title: Integral points in orbits and the Bang-Zsigmondy problem

Abstract: Let f be a rational function that is not conjugate to a powering map, and let a, b be two integers. We show that for any set S of primes, there are finitely many (m, n) such that $f^m(a) - f^n(b)$ is divisible only by primes in S, using the geometry of $\mathbb{P}^1 \times \mathbb{P}^1$. We attempt to put this question in the context of so-called primitive prime divisors suggested by Ingram/Silverman and others. This represents joint work with Corvaja, Gratton, Khoa, Sookdeo, Zannier.

• 13:50-14:40 Yoshinori HAMAHATA (Ritsumeikan University)
Title: Values of Dedekind sums for function fields

Abstract: For the classical Dedekind sum $D(a, c)$, H. Rademacher and E. Grosswald raised two questions:
1. Is $\{(a/c, D(a, c)) \mid a/c \in \mathbb{Q}^*\}$ dense in $\mathbb{R}^2$?
2. Is $\{D(a, c) \mid a/c \in \mathbb{Q}^*\}$ dense in $\mathbb{R}$?
Using the theory of continued fractions, D. Hickerson answered these questions affirmatively. In function fields, there exists a Dedekind sum defined by a given A-lattice (or equivalently, Drinfeld module). The distribution of these values for the Dedekind sum is influenced by the Drinfeld module. In our talk, we answer the analogous questions for Dedekind sums in function fields.

• 14:50-15:40 Hiroshi MIKAWA (Tsukuba University)
Title: Large gaps between Hardy-Littlewood numbers (H. MIKAWA and T. P. PENEVA)

Abstract: It is the well known conjecture of H. Cramér that

$$p_+ - p \ll (\log p)^2$$

where $p < p_+$ stands for consecutive primes. P. Erdős proposed a problem to evaluate its second moment, and A. Selberg showed that

$$\sum_{p \le x} (p_+ - p)^2 \ll x(\log x)^2$$

under the Riemann hypothesis.
This problem has been stimulating many works. For instance, let $\{g\}$ be in ascending order even integers that are representable as the sum of two primes. The Goldbach conjecture is then interpreted as that $g_+ - g = 2$ for all consecutive elements $g < g_+$. Assuming the Riemann hypothesis, Ju. V. Linnik demonstrated that

$$g_+ - g \ll (\log g)^{3+\epsilon}$$

for any $\epsilon > 0$. Here $3 + \epsilon$ may be replaced by 2. It is known that

$$\sum_{g \le x} (g_+ - g)^3 \ll x(\log x)^{300}$$

without any unsettled hypothesis.
It is conjectured by G.H.Hardy and J.E.Littlewood that every large integer, not being a square, is

representable as the sum of a prime and a square. We then call an integer a Hardy-Littlewood number, if it can be written in this manner. Here squares are excluded. We reserve the letter h for Hardy-Littlewood numbers, and write $h < h^+$ for consecutive members. In this talk we shall discuss the second moment

$$\sum_{h \le x} (h^+ - h)^2.$$

• 16:00-16:40 Ken KAMANO (Osaka Institute of Technology)
Title: Analytic continuation of the Lucas zeta function

Abstract:Let $\{F_n\}_{n\geq 1}$ be the Fibonacci sequence and consider the Fibonacci zeta function $\Phi(s) = \sum_{n=1}^{\infty} F_n^{-s}$ for $\Re(s) > 0$. It is known that the Fibonacci zeta function can be meromorphically continued to the whole complex plane and has "trivial" zeros at $s = -4m + 2 \ \ (m = 1, 2, \ldots)$.
In this talk, we investigate the Lucas zeta function defined by $\Phi^{(P,Q)}(s) = \sum_{n=1}^{\infty} U_n(P,Q)^{-s}$ where $\{U_n(P,Q)\}_{n\geq 1}$ is the Lucas sequence associated with integers P and Q. When $(P,Q) = (1,-1)$, the function $\Phi^{(1,-1)}(s)$ is the Fibonacci zeta function $\Phi(s)$. We give the meromorphic continuation of $\Phi^{(P,Q)}(s)$ and prove that it has trivial zeros when $Q = -1$. Moreover, we study Lucas L-functions, which are character analogues of $\Phi^{(P,Q)}(s)$.

• 16:50-17:30 Takao KOMATSU (Hirosaki University)
Title: Sums of products of Cauchy numbers, including generalized poly-Cauchy numbers

Abstract:We investigate sums of products of Cauchy numbers including generalized poly-Cauchy numbers. Recently, the concept of poly-Cauchy numbers has been introduced as a generalization of the classical Cauchy number. A relation among these sums and explicit expressions of sums of two and three products are also given.

. . . . . .